And He Built a Crooked Camera:
A Mobile Visualization Tool to View Four-Dimensional Geometric Objects

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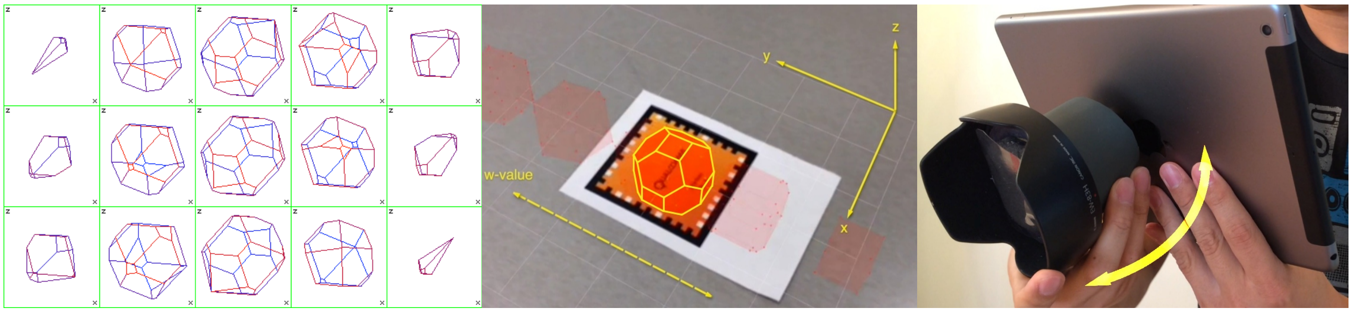

Figure 1: *(left) a traditional approach to view a 4D shape at different points along the fourth dimension. The user must mentally process the 3D shape on 2D paper, and extrapolate between the displayed figures to understand the 4D shape; (middle) our approach allows uses to explore projections of 4D shapes in full 3D with a natural perspective; (right) the fourth dimension, not present in our spatial perception, can be adjusted continuously with a tangible interface.*

Abstract

The limitations of human perception make it impossible to grasp four spatial dimensions simultaneously. Visualization techniques of four-dimensional (4D) geometrical shapes rely on visualizing limited projections of the true shape into lower dimensions, often hindering the viewer's ability to grasp the complete structure, or to access its spatial structure with a natural 3D perspective. We propose a mobile visualization technique that enables viewers to better understand the geometry of 4D shapes, providing spatial freedom and leveraging the viewer's natural knowledge and experience of exploring 3D geometric shapes. Our prototype renders 3D intersections of the 4D object, while allowing the user continuous control of varying values of the fourth dimension, enabling the user to interactively browse and explore a 4D shape using a simple camera-lens-style physical zoom metaphor.

CR Categories: H.5.1 [Multimedia Information Systems]: Artificial, augmented, and virtual realities; H.5.2 [Information Interactions and Presentation]: User interface

Keywords: User interface; mobile interface; high dimensional visualization; visualization

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1 Introduction and Motivation

- *"Maybe they are to you, brother, but they still look crooked to me"*
- *"Only in perspective, only in perspective."*
Robert A. Heinlein's *And He Built a Crooked House* [2]

There are several existing approaches to visualize 4D geometric objects, including Projection (Parallel, Perspective, or Stereographic), Slice, and Depth Cue (Figure 2). Though these techniques can display 4D objects in a relatively straightforward and informative way, they require a steep learning curve and experience to fully understand the components of the visualizations (how the vertices, edges, faces, etc., are related). Ultimately, we argue that the understandability of these techniques is limited, as the geometric representations do not match our natural perception and experience; they, as Heinlein's character complained when observing the design of a 4D house, "look crooked."

One limitation of presenting 4D geometric objects is that they are projected onto 2D surfaces (e.g., paper or a display). Our goal is to capture as much of the original geometric structure as possible while minimizing destruction of perspective or loss of information, although a perfect mapping is impossible due to the limits of human perception. In general, traditional approaches, along with their animated variations rendered in computers (Figure 3), either remove one or more dimensions to show an incomplete geometric structure (e.g., the Slice technique), or error is introduced into a shape's perspective by squeezing 4 dimensions into 2 (e.g., as with all Projection techniques and Depth Cue) (Figure1 left, Figure 2). While such losses of information may be acceptable for simple geometric objects such as a Simplex (4D triangle), more complex shapes lead to larger error or information loss in consequence, hindering the visualization. For instance, the details of the Tesseract (4D cube, shown in Figure 2) are clear, but with traditional visualization techniques, complex shapes such as the 24-cell (Figure 1 left) are much more difficult to parse.

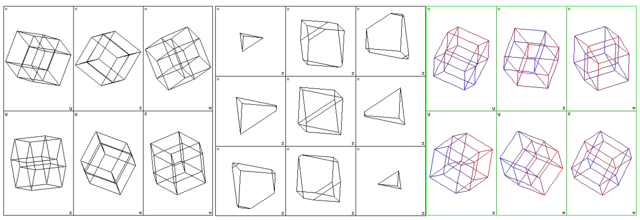

Figure 2: *visualization of a Tesseract with existing methods; (from the left) Parallel Projections, Slices, and Depth Cue [12]*

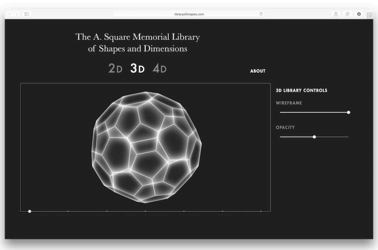

Figure 3: *typical interactive interface of 4D objects, in which all controls upon the hyperspace are operated on a 2D screen, in addition to the complicated camera manipulation*

Our solution for visualizing 4D geometric objects uses a combination of a camera-lens-style physical input (Figure 1 right) and a mobile looking-glass-style display: the mobile display enables users to naturally observe the 3D intersections of the original 4D shape in the higher dimension while benefiting from spatial freedom, i.e. being able to explore it from any arbitrary view angle, while simultaneously exploring the fourth dimension by controlling a physical device. We use a camera metaphor, where a person looks through the camera to view the 4D object, and turns the zoom ring on the lens to shift the visualization along the 4th dimension. For the remaining spatial dimensions, our technique does not require any inherent dimension reduction or perspective distortion, which minimizes the abstraction of the original structure, and viewers are in full control of the exploration. We describe our prototype below.

2 Related Work

Visualizing the geometric structure of different dimensionalities in intuitive and understandable ways has a long history spanning literature [1][2] to geometry [3][4]. Computer graphics and animation techniques [5][6][7] later enabled viewers to interact and manipulate a 4D shape in its digital form [8][9] or even the physical form [10][11]. The contribution of these existing techniques is how they simplify or predigest complicated 4D geometric structures. However, manipulating those 4D shapes by decomposing, unfolding, etc., inevitably incorporates a certain kind of dimension reduction and perspective distortion.

In this paper, we propose a technique that enhances the understandability of 4D shapes by reducing structural abstraction, and leverages users' natural exploring experience.

3 Metonymy and Design intuition

Before diving into the unintuitive 4D world, let us first simplify the story by imagining how people living in a 2D world, as Edwin A. Abbott described in his novel "Flat- land" [1], visualize imaginary 3D geometric objects in an intuitive method. We keep the anatomic basis of the "Flatlanders" (2D people living in a 2D world) as in the original novel, but with 21st century technology.

In Flatland, 1D materials are used to preserve information (paper, book, display screen), and Flatlanders have no difficulty understanding and reasoning about 2D structures, just as we are fully capable of appreciating the 3D world even though our display mediums are usually 2D (paper, book, display screen). Flatlanders have no concept of "up" and "down" along their theoretical z-axis, so when studying 3D geometry, they must look at 2D projections or slices of 3D objects. Conceptually, Figure 4 shows how a sequence of Slice graphs look like in a Flatlander textbook (1D pieces of paper) that introduces a 2-sphere (surface of ball), which is a 3D object and a hyper-object for Flatlanders.

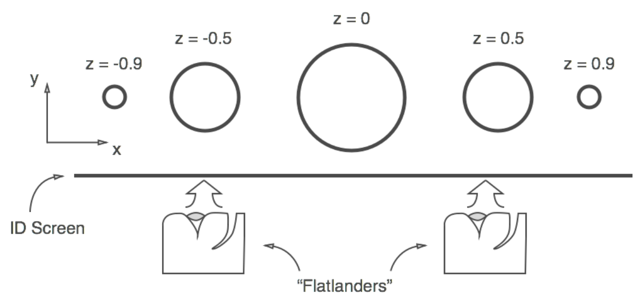

Figure 4: *slice graphs of a 3D object on a Flatland textbook*

By only observing discrete "key frames" (the slices or projections) along the hyper-dimension, Flatlanders may find it hard to mentally reconstruct the continuous geometric shape because they cannot perceive a z-axis. In Figure 4, the key frames are 2D projections of the 3D hyper-object, shown as individual circles, but they need to be further abstracted in order to fit into 1D display mediums in Flatland.

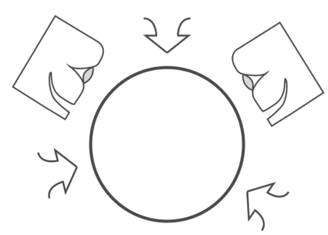

Figure 5: *"Augmented reality"-like visualization in "Flatland"*

Fortunately, in this 2D parallel universe of ours, virtual 2D objects can be illustrated situated at a fix position, allowing Flatlanders to walk around it with 1D "see-through device" and observe it in the Flatlanders' natural 2D perspective, as if a physical 2D entity is being displayed (Figure 5). This idea is similar to "augmented reality", as a virtual object is "pinned" at a fixed position in the space, allowing people to observe it while maintain spatial freedom. However, we use the term "visualization" rather than "augmented reality” because in a hyperspace there is no “reality” for us to augment.

To illustrated 3D hyper-objects, the Flatlanders extract one of the axes, the z-axis in our story, from the hyperspace. At any z-value, the corresponding x-y space contains a 2D intersection of the original 3D shape, just like for a regular 2D object spans in the xy-space, any given x-value corresponds to a y-value. Here any 2D intersection can be illustrated with the aforementioned “augmented reality”-like visualization, providing full spatial freedom to take advantage of Flatlander’s natural perspective, enabling them to explore the 3D shape with a spatial experience that is similar to how they explore their own 2D world every day (Figure 5).

The crucial piece of the puzzle is to design an informative method of letting Flatlanders manipulate the hyper-axis with their hands (or tentacles, depending on what they have), without overlapping or interfering with any physical axis — the x- and y-axis in Flatland — in order to maintain a natural viewing experience. We use a camera metaphor: a photographer moves in space to point-and-shoot, and can adjust the aperture to change the focal depth, considering focal depth as an extra dimension. Flatlanders adopted the metaphor of the camera lens as a physical interface to update the z-value dynamically. Thus, “focusing” the camera lens changes the z-value of the corresponding 2D intersection (circles) of a 3D shape (sphere) in real time (Figure 6).

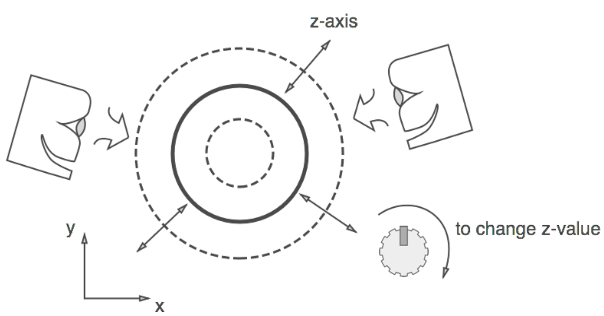

Figure 6: *“augmented reality”-like visualization allows Flatlanders to change the hyper-axis dynamically*

Another goal is maintaining a sense of continuity of the hyper-object, or how the hyper-object will change while browsing along the z-axis. In order to help maintain an overall understanding of the original geometric structure, we display key frames as ghost images at selected z-values (dashed lines in Figure 6). In other words, the Flatland user always knows how the object would change after increasing or decreasing the z-value. This removes the need to constantly rotate the lens, and rotation becomes a tool that provides continuous visualization to link the dots together, helping the Flatlanders understand how the hyper-shape changes in between the key frames.

In summary, by using the aforementioned visualization technique, a hyper-object’s x- and y-axis, the real spatial dimensions in Flatland, are preserved without any perspective distortion and can be observed with their natural spatial freedom. Perception and manipulation of the additional hyper-dimension, the z-axis, is delivered by physical interactions with continuous illustration. In this way, all spatial awareness of the 3D hyper-object is preserved. Also we designed the manipulation of the hyper-dimension to be separated from the fundamental, or “real”, x-y space, so that exploring the hyper-object won’t be confused with updating the z-value. Hence, both of our goals, which are no dimension reduction and perspective distortion, are achieved, and Flatlanders may better understand the essence of a hyper-dimensional 3D object and live ever happily after.

Now let us travel back to our 3D world and apply the same approach; that is, use a similar concept to illustrate a 4D geometric structure, spanning 3 fundamental spatial dimensions plus one hyper-dimension, with no dimension reduction and perspective distortion, in order to provide a more intuitive yet informative way to appreciate a given 4D geometric object.

4 Implementation

We use an iPad Air as the “looking glass” device, and the application is implemented with the Qualcomm Vuforia library. A physical marker is used to situate the center of the rendering in the real world. The device captures both the location and orientation of the marker and renders virtual images correspondingly, as if a physical model has been placed on the marker.

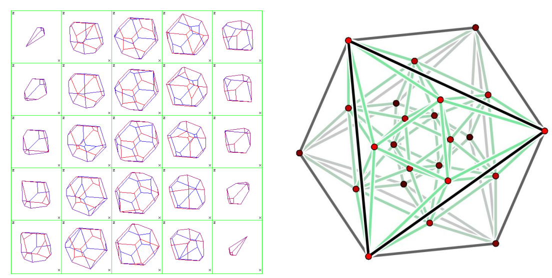

Figure 7: *Traditional approaches to visualize a 24-cell*

To demonstrate the system, we use a 24-cell, a regular polytope in 4D with 24 octahedral cells, 96 triangular faces, 96 edges, and 24 vertices. Due to the complexity of its geometric structure, it is very difficult to understand it with traditional projection techniques. Also, it will be very dense to display all the vertices, edges, and faces in a surface of a limited size (Figure 7).

Similar to our Flatland story, the w-axis in the 4D space is extracted and the user is enabled to adjust its value. Then, the remaining 3 dimensions (x, y, and z) span a regular 3D space. At any given w-value, it is guaranteed by our design that the corresponding 3D intersection can be illustrated without visual distortion, with all the spatial information and freedom maintained (Figure 8).

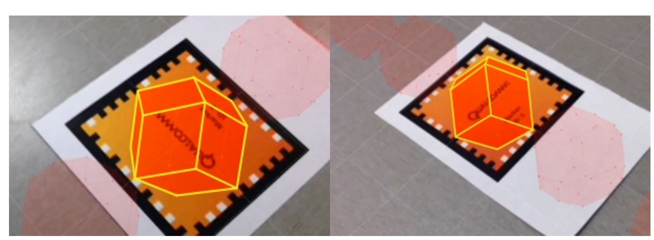

Figure 8: *illustrate 3D projections at any given value on the hyper-axis with a natural perception*

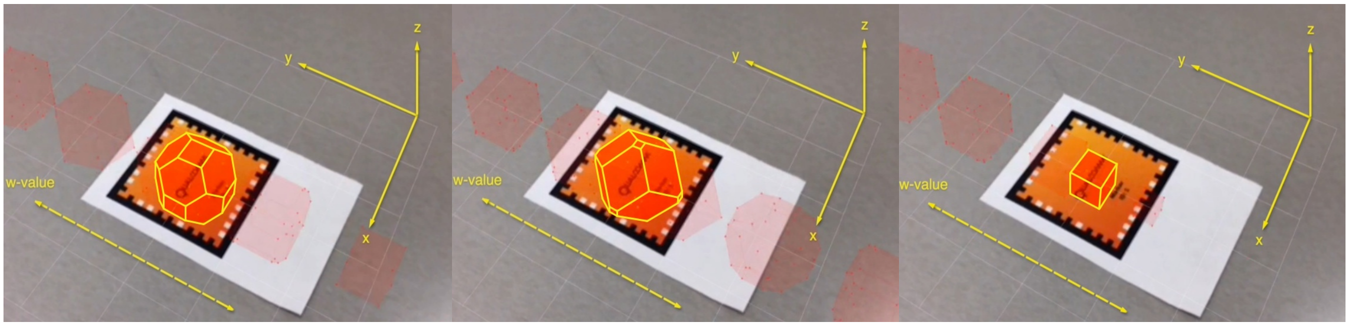

Figure 9: *changing the value along the hyper-axis with the tangible interface continuously and dynamically; key frames are provided as faded 3D intersections to aid the user in the visualization of the continuous hyper-object.*

Moreover, we also constructed a camera lens-looking physical interface with a Phidgets rotation sensor mounted at the back of the tablet, providing the aforementioned pseudo-camera experience of interaction (Figure 1 right). While walking around the visualization of the 3D model situated at the marker, the user can rotate the lens to increase or decrease the w-value, triggering the embedded rotation sensor to update the w-value and the rendered 3D intersection accordingly. As the w-value varies, the smooth real-time transformation of the 3D intersection gradually delivers the idea about the overall geometric structure of the original 4D shape to the user (Figure 9), as Flatlanders see the expansion and contraction of the circle and receive a better understanding of the sphere.

Key frames, represented as ghost images, are also provided at a few selected values (w = -100%, -50%, 0, 50%, 100% of the value), to give the user a hint of how the particular 3D intersection will look after increasing or decreasing the w-value without changing the lens physically. Theoretically, in 4D, these 3D key frames are stacked together like nested dolls, as circles with different radii are positioned at the same center to present key frames of a sphere in the "Flatland" story (Figure 6). However, when many ghost images overlap, it becomes chaotic and difficult to look at (remember, in the "Flatland" story we looked at stacked circles from the third, hyper dimension of their world); thus, we distribute those key frames in a row. The 3D intersection is always situated at the center of the display area, while key frames shift linearly based on the magnitude of the change such that a corresponding key frame coincides with the intersection when both w-values are equal (see ghost images in Figure 9).

5 Critique

We have run a primarily critique session with a small group of participants who have higher education background but not majored in Mathematics. We selected such a target group due to their sufficient knowledge of Mathematics and Geometry but not too much familiarity with hyperspaces.

All participants are able to operate our prototype application independently after a very short training. Participants reported that the "augmented reality"-like observation mechanics are easy to perform and relieve them from tedious and complicated camera manipulation, which is what they commonly deal with on regular display screens. Also, participants understood the camera lens metaphor instantly and had no difficulty operating it, which is the original purpose of our design.

In summary, participants thought the application was "fun", "controllable", and "straight-forward", and helping them to obtain the basic spatial knowledge of 4D geometric structures with experiencing a "less steep learning curve". Moreover, besides improving perceptual easiness, the freedom of maneuvering and applying natural observations made them feel "more confident and masterful", and such a psychological influence is beyond our expectation and we are interested in interpreting it in our future experiments.

A formal study will be necessary for more insight, but even this small critique session suggests the potential of the system as an easy to use, tangible interface to explore hyper-objects.

6 Conclusion and Future Work

We present a mobile prototype visualizing 4D geometric objects using a physical camera-like interface. We consider the following directions for future exploration.

One thread will be applying the same concept to more irregular and complex 4D geometric structures, in addition to the symmetric 4-manifolds that we used to validate our concept in this paper.

Another avenue is higher dimensional visualization. Our method may be scalable to visualizing geometry in 5, 6, or more spatial dimensions, or maybe even a space-time such as the Minkowski space-time continuum. If the simplicity and comprehensibility of our method decreases in these cases, then we need to explore extending the technique to maintain its characteristics in these deeper hyper-dimensions.

We would like to evaluate our prototype via a user study, collecting qualitative and quantitative data related to the intelligibility of the method when observing and studying a 4D geometric object or structure compared to traditional visualization techniques, or even the pure text-base notations that only make sense to experts. Furthermore, it will be interesting to observe two participant groups with different expertise level use our tool, one with sufficient amount of mathematical knowledge and one without, and see whether our interface provides additional insight to either of the groups.

In summary, we presented a new method to illustrate and interact with 4D geometric objects. We carefully designed the visualization to provide the user with a familiar visual representation of the 3D intersection of the object without distortion, enabling free spatial exploration, and allowing the fourth hyper-dimension to be controlled and manipulated by the user who is continuously and dynamically updating the 3D intersection.

We hope that our method and prototype could set the stage and inform future research on this topic, potentially bringing this design concept to help illustrate high-dimensional scientific information.

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